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MATHEMATICAL MODELING OF EVAPORATIVE COOLING OF WATER FILMS IN WATER-COOLING TOWERS

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A mathematical model of evaporative cooling of water films flowing down vertical guards in a chimney-type water-cooling tower is developed. Results of a qualitative analysis of the nonlinear mathematical model are reported. Data obtained from a numerical solution of a boundary-value problem for a system of ordinary differential equations are presented in the form of graphs.

Introduction. Evaporative cooling of circulating water in chimney-type water-cooling towers of a heat and electric power plant [1] cannot be accomplished efficiently without organizing film flows. Present-day water-cooling towers have the following parameters: a tower height of about 100 m, a diameter of the base of ~60 m, a water flow rate of about 30,000 ton/h. Water cooling in cooling towers proceeds mainly due to evaporation of water at a temperature of 25–40°C flowing by gravity down sprinkler guards in the form of films (evaporation of 1% of the water decreases its temperature by approximately 6°C) [2].

The efficiency of evaporative cooling of water films depends on numerous parameters, the most important of which are the specific flow rate Q_w and the initial water temperature T_{w0}, the temperature T_{m0} and moisture content ψ of the vapor-air mixture at the tower inlet, and the air velocity u, which depends on the height of the cooling tower and the degree of air heating. Geometric parameters, namely, the height of the guards H and the distance between guards d, also play an important role. The physics of evaporative cooling is well studied, but quantitatively, the interaction of many physical processes can be calculated presently only by methods of mathematical modeling. The goal of the present work is to investigate the influence of the main parameters on the efficiency of evaporative cooling, which is important for development of both methods of heat-transfer intensification and the physical principles of systems of control of cooling towers [3, 4].

It should be noted that heat exchange of drops with an ascending vapor-air mixture, investigated in [5], plays an important role in water cooling in cooling towers.

The influence of various disturbing factors on the surface parameters in film flows is described in detail in [6].

Mathematical Model. Consideration is given to evaporative cooling of a thin layer of a viscous fluid running down two vertical guards. It is assumed that air goes up between the guards with a constant velocity. As the air moves upward, it is heated and the concentration of water vapor in it increases, thus slowing down the intensity of heat exchange with the descending water film. The air velocity between the guards depends on the change in the vapor-air mixture density in the cooling tower, determined mainly by the degree of air heating. The interaction of the water and air flows considered in the present work represents a "unit cell" of the chimney-type cooling tower if we restrict ourselves to film flow of water. Figure 1 depicts a schematic of the investigated problem.

After the standard procedure of averaging, in the transverse direction, of the profiles of the film temperature and velocity, the vapor-air mixture temperature, and the water-vapor density the system of equations describing the processes of heat and mass exchange between the film vertically running down and the ascending vapor-air mixture has the form

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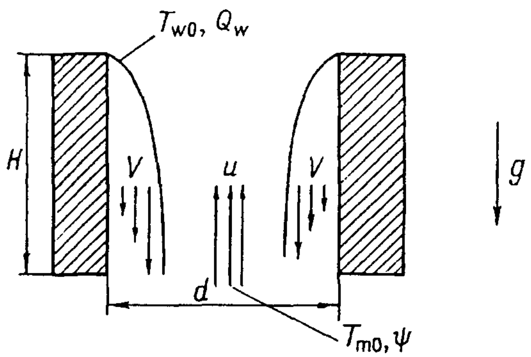


Fig. 1. Schematic of the water and air flows.

$$\frac{dh(z)}{dz} = -\frac{\gamma(\mathrm{Re})(\rho_s(T_w(z)) - \rho(z))}{\rho_w V}, \tag{1}$$

$$\frac{dT_w(z)}{dz} = \frac{\alpha(\mathrm{Re})(T_m(z) - T_w(z))}{c_w \rho_w h(z) V} - \frac{\gamma(\mathrm{Re})(\rho_s(T_w(z)) - \rho(z))}{\rho_w h(z) V}\left(\frac{r}{c_w} + T_w(z)\right), \tag{2}$$

$$\frac{dT_m(z)}{dz} = -\frac{2\alpha(\mathrm{Re})(T_w(z) - T_m(z))}{ud(\rho_a c_a + \rho c)} + \frac{T_m}{\rho_m}\frac{d\rho}{dz}\left[1 + \frac{\rho_a(c - c_a)}{\rho_a c_a + \rho c}\right], \tag{3}$$

$$\frac{d\rho(z)}{dz} = -\frac{2\gamma(\mathrm{Re})(\rho_s(T_w(z)) - \rho(z))}{ud}. \tag{4}$$

Equation (1) describes the change in the film thickness $h(z)$ as a consequence of its evaporation; Eq. (2) determines the change in the water temperature $T_w(z)$ averaged over the film cross section due to contact with cold air and evaporation; Eq. (3) allows calculation of the change in the temperature of the vapor-air mixture $T_m(z)$ averaged over the cross section between the guards; Eq. (4) describes the change in the water-vapor density $\rho(z)$ in air.

The boundary conditions for the system of equations (1)-(4) are written as follows: at $z = 0$ the initial thickness and temperature of the water film are prescribed:

$$h = h_0, \tag{5}$$

$$T_w = T_{w0}; \tag{6}$$

at $z = H$ the vapor-air mixture temperature and the water-vapor density in the mixture at the cooling-tower inlet are prescribed:

$$T_m = T_{m0}, \tag{7}$$

$$\rho = \rho_s(T_{m0})\psi. \tag{8}$$

Thus, for the system of ordinary differential equations (1)-(8) we have a nonlinear boundary-value problem.

In Eqs. (2), (3), $\alpha(\mathrm{Re})$ is the heat-transfer coefficient, determined as follows [7]:

$$\alpha(\mathrm{Re}) = \frac{0.324\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.33}\,\lambda}{x}. \tag{9}$$

With account for the geometry of the problem the Reynolds number is determined [7, 8] as

$$\mathrm{Re} = \frac{x\rho_a (u + V)}{\mu}, \tag{10}$$

where $(u + V)$ is the relative velocity of the air flow past the water film; x is the coordinate reckoned from the lower edge of the guards. It is pertinent to note that a rigorous description of our problem would require the use of three Reynolds numbers: for the film flow, for the vapor-air mixture flow between the guards, and for the processes of interphase heat and mass exchange. We have used one Reynolds number, which remains after averaging all the corresponding parameters over the film and vapor-air mixture thicknesses.

Using the analogy between the dependences of the heat-transfer (the Nusselt number $\mathrm{Nu} = \alpha x/\lambda$) and mass-transfer (the Sherwood number $\mathrm{Sh} = \nu x/D$) numbers on the Reynolds number we can determine the mass-transfer coefficient for a laminar flow past a thin water film as

$$\gamma(\mathrm{Re}) = \frac{0.324\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.33}\,D}{x}. \tag{11}$$

The temperature dependence of the diffusion coefficient of water vapor D was taken from [9] and was taken into account in numerical modeling.

The velocity of the descending water film averaged over the cross section and entering the system of equations (1)-(4) is determined in a laminar approximation [8] as

$$V = (g^2 Q_w / 18\,(\nu_w)^2\,\rho_w)^{2/3}.$$

It should be emphasized that for chimney-type evaporative cooling towers the condition $H \gg d \gg h$ is fulfilled, which allows the mathematical formulation of the investigated problem to be simplified considerably.

With satisfactory accuracy the vapor-air mixture velocity u in a chimney-type cooling tower is described by an expression stemming from the continuity equation for the vapor-air mixture flow through the cooling tower:

$$u = (R_1/R_0)^2\,((2gH_t\Delta\rho_m)/\rho_m)^{0.5}\,k,$$

where R_1 is the throat radius; R_0 is the sprinkler radius of the cooling tower; in calculations the value of the empirical coefficient k is assumed to be equal to 0.5 [10], and the change in the vapor-air mixture density $\Delta\rho_m$ is determined in the course of solving the problem and takes into account the change in the temperature and composition of the mixture at constant atmospheric pressure.

The boundary-value problem for the system of nonlinear ordinary differential equations (1)-(8) was solved by the shooting method [11]. For numerical solution of the system of differential equations use was made of the Runge–Kutta method of fourth order. The accuracy was monitored with the aid of the discrepancy criterion Σ:

$$\Sigma = \left(\left(\frac{T_m(H) - T_{m0}}{T_{m0}}\right)^2 + \left(\frac{\rho(H) - \rho}{\rho}\right)^2\right)^{1/2}.$$

The shooting method was terminated upon fulfillment of the condition $\Sigma < 10^{-2}$. A subsequent increase in the accuracy exerted practically no influence on the change in the water temperature and other parameters during calculations.

Calculation Results. The efficiency of operation of a chimney-type cooling tower is evaluated by the thermal efficiency η determined by the expression [1]

$$\eta = \frac{T_{wf} - T_{w0}}{T_{lim} - T_{w0}}, \tag{12}$$

where T_{wf} is the temperature of the outflowing water; T_{lim} is the limiting temperature of water cooling in evaporative cooling for a given temperature of the inflowing vapor-air mixture $T_m(z = H)$ and relative moisture content ψ. The value of T_{lim} is determined from the equation

$$\rho_s (T_{lim}) = \rho_s (T_m) \psi .$$

From the physical viewpoint T_{lim} is equal to the wet-thermometer temperature.

Before passing to the results of numerical calculations, we qualitatively analyze the mathematical model (1)-(8) [12]. For simplicity let us start with an analysis of Eq. (4). We integrate it with respect to the coordinate z from 0 to an arbitrary coordinate z_1. Then the left-hand side of Eq. (4) is equal to $\rho(z_1) - \rho_f$, where ρ_f is the water vapor density in the vapor-air mixture upon leaving the space between the guards, while the right-hand side is

$$- 2\cdot 0.0324 \, \mathrm{Pr}^{0.33} \frac{D}{du} \left[\frac{\rho_a (u + V)}{\mu}\right]^{0.5} \int_0^{z_1} \frac{\rho_s (T_w (z)) - \rho (z)}{(H - z)^{0.5}} dz . \tag{13}$$

After approximate calculation of the integral in (13) we obtain

$$(\rho (z_1) - \rho_f) \propto - \frac{D}{d} \left[\frac{\rho_a H (1 + V/u)}{\mu u}\right]^{0.5} (\rho_s (T_w (0)) - \rho) (1 - \sqrt{1 - z_1/H}) . \tag{14}$$

With account for the exponential temperature dependence of the saturated-vapor density expression (14) can be written in the form

$$(\rho (z_1) - \rho_f) \propto - \frac{D}{d} \left[\frac{\rho_a H (1 + V/u)}{\mu u}\right]^{0.5} \rho_s (T_w (0)) (1 - \sqrt{1 - z_1/H}) . \tag{15}$$

For small z it is easy to see from (15) that the density profile of the water vapor with respect to the height of the guards is linear. Here, the water flow rate influences the water-vapor density via the ratio of velocities V/u:

$$(\rho_f) \propto \frac{D}{d} \left[\frac{\rho_a H (1 + V/u)}{\mu u}\right]^{0.5} \rho_s (T_w (0)) . \tag{16}$$

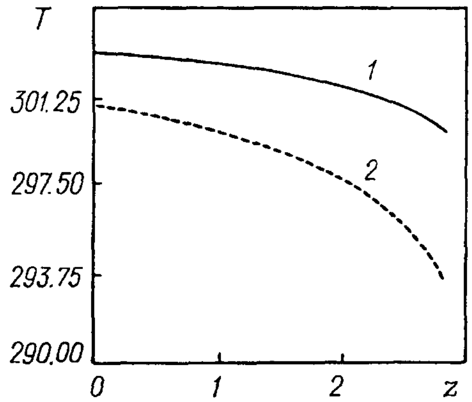
Note that from physical considerations $\rho_f < \rho_s(T_w(0))$. From expression (16) it follows that the main factors determining the density of the water vapor in the vapor-air mixture outflowing from a slot are the initial water temperature and the quantity D/d expressing the ratio of the geometric parameters of a "unit cell" of the cooling tower.

Similar estimates for Eq. (3) lead to the following approximate expression for the temperature change of the vapor-air mixture:

$$(T_m (H) - T_{a0}) \propto \frac{\lambda}{d (c_a \rho_a + c\rho)} \left[\frac{H (1 + V/u)}{\mu u}\right]^{0.5} [T_{w0} - T_{m0}] , \tag{17}$$

From (17) it follows that the temperature drop of the vapor-air mixture ΔT_m after evaporative cooling is directly proportional to the difference of the initial temperatures of the water and the air surrounding the cooling tower and inversely proportional to the distance between the guards and depends on the specific water flow rate via the parameter V. Note that the dependence of ΔT_m on the length of the guards is weak ($\Delta T_m \sim \sqrt{H}$).

Now we apply similar qualitative estimates to an analysis of Eq. (2). As a result of approximate integration, we obtain



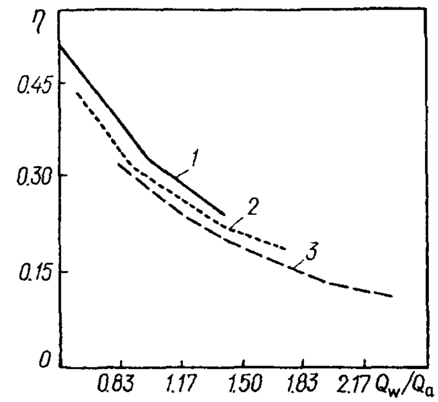


Fig. 2. Temperature profiles of water and air: 1) water temperature; 2) temperature of the vapor-air mixture; T_{w0} = 303 K; air temperature at the inlet T_{a0} = 293.15 K; ψ = 0.7.

Fig. 3. Thermal efficiency of the cooling tower η versus ratio of the mass flow rates of water and air Q_w/Q_a: 1) $T_{w0} = 35^oC$, 2) 30, 3) 25. T_{a0} = 293.1 K; ψ = 0.7.

TABLE 1. Influence of the Relative Humidity of Air ψ on the Thermal Efficiency η and the Temperature Drop of Water

ψ	0.5	0.6	0.7	0.9
η	0.24	0.257	0.275	0.293
$T_{w0} - T_f$	5.1	4.7	4.4	4

$$(T_{w0} - T_w(H)) \propto \frac{\mathrm{Re}^{0.5}}{c_w Q_w} [\lambda (T_{w0} - T_{m0}) + \mathrm{Dr} (\rho_s (T_{w0}) - \rho)] . \tag{18}$$

After simple transformations with account for (12), a qualitative estimate for η follows from (18):

$$\eta \propto \frac{Q_a}{Q_w} \frac{H}{d} \frac{\left(1 + \frac{V}{u}\right)}{\mu c_w \mathrm{Re}^{0.5}} \left[\lambda \frac{T_{w0} - T_{m0}}{T_{w0} - T_{\lim}} + \mathrm{Dr} \frac{\partial \rho_s}{\partial T} \bigg|_{T=T_{\lim}} \right] . \tag{19}$$

The specific flow rate of the air Q_a is determined as $Q_a = d\rho_a u$. From (19) it follows that η depends on many parameters, and its value depends on both the water evaporation and the usual heat transfer.

Results of numerical solution of system (1)-(8) are presented in Figs. 2, 3. Figure 2 shows temperature profiles of water and air calculated using the mathematical model described. It is pertinent to note that they are quite consistent with the results of the qualitative analysis reported above. It is seen that the processes of heat and mass transfer are most pronounced in the lower part of the guards. From the physical viewpoint this is related to the smaller thickness of the boundary layer in the vapor-air mixture above the water film [12]. With increasing thickness of the boundary layer the intensity of heat and mass transfer processes decreases. As calculations made in the present work showed, at low flow rates of water the temperature profile of the vapor-air mixture approaches a linear one.

Figure 3 represents the thermal efficiency η of the cooling tower as a function of the ratio of the mass flow rates of water and air obtained as a result of numerical experiments for different temperatures of the supplied water. A comparison of the data presented reveals that the main factor influencing η is the ratio of the mass flow rates of water and air. The nonlinear character of the curves in Fig. 3 is attributable to the fact that the flow rate of air in the cooling tower depends on the temperature difference of the mixture and the surrounding air (17). This difference depends, in turn, on the flow rate of water. The dependence of η on the other parameters influencing the evaporative cooling is expressed well by formula (19).

Table 1 shows the dependence of the thermal efficiency η of the cooling tower and the temperature drop of the water on the relative humidity of the air ψ. In the calculations it was assumed that $T_{a0} = 20^{o}C$, $T_{w0} = 30^{o}C$, the specific flow rate of the water $Q_w = 0.032$ kg/(m·sec), $p = 740$ mm Hg, $H = 3$ m.

From the calculated results provided in Table 1 it follows that η is a weakly changing function of the air humidity. Only for the case $Q_w/Q_a < 1$ is η independent of the moisture content, which agrees with [5].

Conclusion. A mathematical model of evaporative cooling of water films in cooling towers is constructed. The model is a boundary-value problem for a system of four nonlinear ordinary differential equations that allow determination of the changes in the film thickness over its length, in the water temperature averaged over the film thickness, and in the vapor-air mixture temperature and the water vapor density in the mixture.

A qualitative analysis of the mathematical model has made it possible to obtain approximate formulas (16)-(19) relating the geometric dimensions, thermophysical properties, and flow rates of the heat-transfer agents in evaporative cooling. The results of the numerical calculations point to the reliability of the qualitative analysis.

As a result of numerical experiments, it was shown that the thermal efficiency η of the cooling tower is a nonlinear monotonically decreasing function of the ratio of the mass flow rates of the water and the air Q_w/Q_a. Moreover the air flow rate depends on both the geometric parameters (the tower height, the distance between the guards, and their length) and the specific flow rate and temperature of the water. The values of η calculated for data obtained on a commercial cooling tower are quite consistent with known values of η found by processing results of full-scale measurements [10].

It is of interest that the water vapor density in the vapor-air mixture outflowing from "a unit cell of the cooling tower" formed by the guards amounts to ~(0.9–0.95) of the saturated-vapor density at the mixture temperature. Here, the mixture temperature is $T_f = T_{m0} + (T_{w0} - T_{m0})K_t$. The value of K_t was determined in the course of solving system (1)-(8), and in our calculations it was ~(0.65–0.8). Depending on the atmospheric conditions near the cooling tower, in all, (0.5–1)% of the discharge of water evaporates for a length of the guards of 3 m. Calculations showed that a subsequent increase in the dimensions of the guards practically does not influence the effectiveness of evaporative cooling due to effects of saturation of the vapor-air mixture. The saturation effect exerts an adverse influence on the operation of evaporative-cooling facilities, and the undertaking of measures against it is an important engineering problem.

It should be noted that water cooling proceeds mainly owing to thermal-energy losses to evaporation (65%).

As our numerical experiments showed, the thermal efficiency of the cooling tower η that is found without account for the increase in the temperature and the moisture content in the ascending air flow exceeds the value of η that accounts for these changes by almost a factor of two. This result is important both for evaluation of the maximum efficiency of a cooling tower and for investigation of time-periodic irrigation regimes [13].

It seems reasonable to refine the mathematical model of evaporative cooling of water films. This can be done, foremost, by not adopting the temperature of the vapor-air mixture and the water-vapor density averaged over the flow cross section and by taking into account the turbulent nature of the vapor-air mixture flow. Results of investigations along this line will be published.

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NOTATION

T_{wf}, final water temperature; T_f, final temperature of the vapor-air mixture; Q_a, mass flow rate of air, kg/(m·sec); h, thickness of the water film, m; V, velocity of the descending water film averaged over the cross section, m/sec; g, free-fall acceleration, m/sec^2; z, coordinate along the guards reckoned from the upper edge, m; x, coordinate along the guards reckoned from the lower edge, m; c_w, specific heat of water, J/(kg·K); c_a, specific heat of air, J/(kg·K); c, specific heat of water vapor, J/(kg·K); r, heat of vaporization, J/kg; ψ, relative humidity of air; ρ_m, density of the vapor-air mixture, kg/m^3; ρ, density of water vapor, kg/m^3; ρ_f, density of water vapor at the point $z = 0$, kg/m^3; ρ_s, density of saturated water vapor, kg/m^3; ρ_a, air density, kg/m^3; ρ_w, water density, kg/m^3; γ, mass-transfer coefficient, m/sec; λ, thermal conductivity of air, W/(m·K); α, heat-transfer coefficient, W/(m^2·K); μ, dynamic viscosity of air, kg/(m·sec); ν, kinematic viscosity of air, m^2/sec; ν_m, kinematic viscosity of water, m^2/sec; H_t, height of the cooling tower, m; k, empirical coefficient. Subscripts: m, vapor-air mixture; 0, initial value of the parameter; w, water; a, air; f, final value of the parameter; t, parameter of the cooling tower.

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